Tractatum de Modi Sephiratorum

Kosmorsky

"Neque porro quisquam est qui dolorem ipsum quia dolor sit amet, consectetur, adipisci velit."

-- Cicero

(You can skip the intro, to page two.)

Several years ago I remember I had a particularly unusual dream, consisting of strange music in which a "sixth" somehow sounded like a "fifth". Playing sixths on the piano assured me this was nonsense (before my introduction to microtonality), so I dismissed it as the heat playing tricks on my brain during the nap which it had compelled. Only now, about a month or two after discovering these Modi Sephiratorum mathematically, I remembered this event and that it could plausibly have a connexion.

While I consider myself something of a microtonalist, I prefer to try to extend tradition than escape it. Good music is good music. In particular the baroque aesthetic draws me, partly because it intertwines and balances melody and harmony with perfection rarely attained since. I must point out, even contemporary music is gradually reinstating the ideal, in so many ways I would diverge irreparably to mention them.

The aspects that seem most important to my ideal are polyphony, with its ground in just harmony, and modal linearity, which have abundant implications. I would consider it a great accomplishment to find another musical scheme which handles these properties as naturally as the 3 and 5-limit diatonicism of the past, with any luck even more colourfully, and I see that potential strongly in the scales exhibited here.

The most obvious place to begin a search is by stacking other harmonics within an octave frame, which I prefer to call a "modal chain". The 3rd harmonic bears the fruit of pythagorean scales, which can be tempered flat, leading toward the 5th harmonic in meantone, and other harmonics when extended. It is from this framework that the terminology of "fifths" etc. which I think are often abused in a microtonal context.

These 'fifths' can be tempered otherwise to other results, but for the sake of argument let's move on. Modal chains of the 5th harmonic result in jagged scales with large gaps and no generally redeeming harmonizations to speak of. The seventh harmonic, bears a natural affinity to the 11th harmonic, and the heptatonic scale which unites them follows the form LsLsLsL and is aptly named "Orgone" by Andrew Heathwaite. It has a fascinating, very capable alternate tonality, and forms of it can be found in 11edo, 15edo, and most accurately 26edo, but to put it simply, I was looking for something else.

Mind you! These aren't the only possible tunings up to this point, there are an infinitude of them, equal divisions, just intonation, etc. all of them important. The Bohlen Pierce scale and Paul Erlich's decatonic scale deserve special note, but neither are constructed from chains per se. In fact, the BP scale is derived from two harmonic directions instead of one, plus the third harmonic which is equivalent instead of the octave. The decatonic scale is a convenient, symmetrical, though somewhat nonlinear, gamut of 7 limit harmony, like the meantone of the 7 limit.

But you know what, I like the 13th harmonic. It sounds good.

So what happens when the 13th harmonic is made the foundation of harmony and modality? A neat variety of Moment of Symmetry scales: a tetrad, heptatonic, and decatonic scales are the results. The decatonic scale formed from a just 13th very, very nearly closes to an octave, and the 10 tone equal temperament offers a nearly perfect approximation.

Curiously now, you may notice that mildly flattening that 13th harmonic, other consonances are arrived at straightaway, namely the 21st and 17th harmonics. The first MOS produced this way itself functions as a 16:17:21:26 tetrad! In fact, probably because 13/8 is an approximation of phi, by tempering the 13th harmonic suitably, a good portion of the Fibonacci sequence is approximated as harmonics!

23 edo does this remarkably well, and approximates the fibonacci harmonics up to 4181/4096 within 17 cents of accuracy, and is very accurate at the beginning of the sequence diverging gradually, whereas 36edo, having a closer approximation of phi, has slightly worse values for the lower fibonacci harmonics, and more accurate higher terms. How directly perceptible harmonics upwards of the 610th are is an open question, probably not much, but it maybe the quality of a better temperament of the golden ratio might be detectable itself, as well as the possible value of fibonacci harmonics as a framework for the temperament and arrival at the lower harmonics themselves.

For the record, I didn't much care for the scale built on phi directly as the value for “13/8”, but it might be quite useful itself (it could grow on me too), and perhaps scales built from roots of phi, or multiplications of the phi-scale by roots of two, might be a way to unleash those prime factors which the fibonacci sequence does an interesting job with. The curious might consult the sections of this page relating fibonacci sequence and prime numbers: http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/fibmaths.html#fibprimecarm

But suppose we take a step back and use that first tetrad as our basic consonance, akin to the major chord of common practice. 23-edo favors the lower fib. harmonics anyway, so until further notice we'll use that as a model for our "meantone". The next moment of symmetry appears at seven notes, in which there are to be found four tones with major tetrads, and three tones with tetrads with altered tones (one tetrad has one altered tone, the next has two, and in the third all but the unison is minor, somewhat like a diminished chord).

These heptatonic modes are the ones discovered independently by Andrew Heathwaite, and they exhibit the interesting property that the chord is built on every other tone of the scale, but in descending rather than ascending order, so in fact while the minor tones are higher in pitch, they are still in fact "minor"/"diminished". It's completely upside down! So I choose to notate them in descending order though it's bound to confuse a few people.

This suggests certain musical arrangements, such as using bass melodies with treble harmonies, which is something ancient Greek music did if I remember correctly. In fact, I might relate an anecdote, how just last night as I was playing around with the 17-edo notes directly above and below the tonic, the one higher in pitch started to sound like 'down' and the one lower like 'up' in a peculiar reverse of the common perception.

As it is the heptatonic scale is both tonal and quite xenharmonic, but what about the decatonic scale which is the next level up in complexity? Ten tones is not too much of a stretch for the human brain. The chord tones are now two tones removed, still descending, and the "minor" chords of the heptatonic scale become the "suspensions" of the decatonic scale; and there are three diminished chords (one with one diminished tone, one with two and another with three) at the end of the modal chain.

With the basic structure of the scale established, some terminology might be helpful at this point. The modes are named according to the Sephiroth, with Keter something like Lydian mode, being all major, and Malkuth like Locrian mode, all minor. I would propose a six line staff for the decatonic scale such that each natural tone gets a line or a space. Cleffs I have not worked out, but for now a little "K" centering on Keter suffices for my modal purposes.

As for the intervals themselves, the unison I'll call a first, and the 2/1 interval I'll call the "undecave" by analogy to the octave. Because the scale is labelled in descent (upside down) the 13th harmonic, which is the main structural interval ("like a fifth") is the fourth degree. A major interval is derived from the part of a modal chain following the tonic, and a minor interval // preceeding the tonic. The term diminished is applied to the chord tones derived from exhaustion of the 10-note scale, and minor refers to general exhaustion, whereas suspensions from exhaustion of the 7 note scale.

Chord tones are the first, fourth, seventh, and tenth degrees down from a given note, while suspensions (related to the heptatonic scale, see above) are found on the third, sixth, and ninth degrees. And don't forget that intervals can be diminished or major, according to place in the modal chain. See the charts below, I urge you.

An interesting note: instead of two tetrachords, the natural structure of the scale tends to three tetrachords, such as A-B-C-D; D-E-F-G; G-H-I-J; A... like so. The heptatonic scale groups analogously to triplets. The break defines the mode, and can be used for special melodic emphasis.

The circle of fourths (tm) goes like this for Keter:

I - IV - VII - X - III(m9) - VI(m6) - IX(m3) - ii(m10) - v(m7) - iix(m4) - I...

For other modes, shift the roman numerals. Chord symbols first relevant minor, the rest implied.

Well that's about as developed as theory is for these 'uns. Continuing to stack on the 13th harmonic adds three notes. The 4-note MOS would be the major tetrad, the heptatonic scale for simple underlying melodic structures, the decatonic scale extends it to full melody, then 13 and 16 tone 'chromatic' scales, and beyond if need be.

On chord voicings, I find that the tonic is discernible as a kind of sense, much like in familiar diatonicism, which is only modulated by voicing. But here the harmony is a little more complex, and it can be a bit more vague. The root doesn't have to be on the bottom, but is best annunciated there sometimes. A particularly simple and pleasing harmony comes from just stacking the 13/8 interval to give very wide chords, Polyphony works very well too, the voices can sound very independent, while harmonizing nicely and outlining chord progressions, which is what tonality is all about.

Hearing the 21st harmonic as a consonance instead of as a 4/3 can be a difficult habit to break, but having the other chord tones in there helps a lot. The 17th harmonic can be a shocking consonance to, but you quickly get used to it. Something really nifty, 144, which is a fibonacci number and therefore an harmonic in the MS, octave reduces to to the familiar 9th harmonic, which is of course consonant too, but you don't want it to dominate the texture all the time. Horn and string timbres work very well, I haven't tried human voice but I think that would sound right.

A keyboard could be easily designed, one with seven white keys and six accidentals, for an (un)equal 13 tone scale, would be convenient, as would one with 10 thin white keys and six or more black (allowing the fingering 1-2-3-4-5-1-2-3-4-(5) haha). The former ought to be easy enough to retroengineer, but the latter would be rather impressive to exhibit.

While I mentioned a few tunings already: 10 edo, 23 edo, 36 edo, and Phi itself, others deserve mention. 13 edo is the simplest to distinguish large and small steps, even though it's tempered a bit heavily; a 13 tone subset or even a 13 tone well temperament may also be of value. 16edo, 19 edo and 22edo equate 8/5 with 13/8, the scales sound jagged but might be useful. 22 at least has a good 17/16 outside the modes, and can be played schismatically. In this way, 34 edo can be played schismatically for 5 limit music as well as the MS, although with a 13th harmonic 7 cents sharp, it has some interesting stuff going on of its own, like the 17edo decatonic scale which approximates 4/3 instead of 21/16 ... 33 edo is another fascinating choice of tuning, which to a degree can play 19 and higher limit harmony and is tempered every which way. 43 edo, which is approximately 1/5 comma meantone, has a better 13th harmonic in return for slightly worse 17th etc. The notable 53 edo also contains the MS.

A short list of L/s sizes of MS for smaller EDO:

L=1 s=1 10edo

L=2 s=1 13edo

(L=3 s=1 16edo)

L=3 s=2 23edo

(L=4 s=1 19edo)

L=4 s=3 33edo

(L=5 s=1 22edo)

(L=5 s=2 29edo)

L=5 s=3 36edo

L=5 s=4 43edo

(L=6 s=1 25edo)

L=6 s=5 53edo

L=7 s=6 63edo

L=7 s=5 56edo

L=7 s=4 49edo

etc.

The charts:

													harmonic		
L	0	Keter		*m*	*m*	*m*	*m*	*m*	*m*	*m*	*m*	*m*	harmonic		Tonic, Unison
s	7	Hod	M		**M**	M	*m*	**M**	M	*m*	**M**	M	233		Major / minor 4th
s	4	Gevurah	M	*m*		**M**	*m*	*m*	**M**	*m*	*m*	**M**	55		7th
L	1	Chokmah	**M**	*m*	*m*		*m*	*m*	*m*	*m*	*m*	*m*	13		10th
s	8	Yesod	M	**M**	M	M		**M**	M	*m*	**M**	M	377		3rd
s	5	Tiferet	M	*m*	**M**	M	*m*		**M**	*m*	*m*	**M**	89		6th
L	2	Binah	**M**	*m*	*m*	**M**	*m*	*m*		*m*	*m*	*m*	21		9th
s	9	Malkuth	M	**M**	M	M	**M**	M	M		**M**	M	610	This table compares the intervals of the ten modes, in terms of major (harmonic) and minor (subharmonic). The modes descend and are so labelled ie.	
s	6	Netzach	M	*m*	**M**	M	*m*	**M**	M	*m*		**M**	144		
s	3	Chesed	**M**	*m*	*m*	**M**	*m*	*m*	**M**	*m*	*m*		34		
	0	Keter		"Undecave"									$1/2^n$	Higher Pitch Lower Pitch	

#	MS	Heptatonic modes found in
0	Keter	led
1	Chokmah	led, jwl
2	Binah	led, jwl, fish
3	Chesed	led, jwl, fish, bish
4	Gevurah	jwl, fish, bish, kleeth
5	Tiferet	fish, bish, kleeth, gil
6	Netzach	bish, kleeth, gil, dril
7	Hod	kleeth, gil, dril
8	Yesod	gil, dril
9	Malkuth	dril

The heptatonic modes (Heathwaite)

L	0	led
s	4	kleeth
L	1	jwl
s	5	gil
L	2	fish
s	6	dril
s	3	bish
	0	led

* These are also in desending form, whereas Heathwaite has them ascending.